DEFORMABLE REGISTRATION WITH SPATIALLY VARYING DEGREES OF FREEDOM CONSTRAINTS

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ABSTRACT

Intra-subject deformable registration applications, such as longitudinal analysis and multi-modal imaging, use a high degree freedom deformation to accurately align soft tissue. However, smoothness constraints applied to the deformation and insufficient degrees of freedom in the deformation may distort the more rigid tissue types such as bone. In this paper, we present a technique that aligns rigid structures using rigid constraints while aligning soft tissue with a high degree of freedom deformation.

***Index Terms*—** Deformable image registration, optical flow, anatomy-specific constraints, volume preservation.

1. INTRODUCTION

Intra-subject image registration is a powerful tool for estimating coordinate frame mappings in longitudinal sequences, time varying (4D) acquisitions, interventional imaging, and multi-modal imaging. The coordinate frame mappings allow the clinician to fuse information coming from differing sources, imaging anatomy, function or select pathology. When imaging the thorax or abdomen, deformable registration is needed to accurately estimate the coordinate frame mappings between soft tissue types in order to correct for shape change due to respiration. However, the abdomen and thorax contain a variety of tissue types with widely varying rigidity. Individual structures, such as bone, predominantly move rigidly with changes in patient position and posture while surrounding soft tissue deforms non-rigidly. While deformable registration is capable of modeling rigid motion, most deformable methods use either too few degrees of freedom in modeling the coordinate frame mapping, making it difficult to accurately model proximal rigid and nonrigid motion, or use too many degrees of freedom in modeling the coordinate frame mapping, resulting in an over-fitting of sensor variations on rigid structures and unrealistic local minima mappings.

A variety of approaches have been published to address both rigid and nonrigid motion leveraging image registration. Little *et al.* [1] present a thin-plate spline method to define nonrigid coordinate frame transformations where portions of the transform are constrained to be linear (affine). Staring *et al.* [2] adaptively filter a deformation field to apply rigidity constraints during regularization. Ruan *et al.* [3] regularize a dense deformation field such that the Jacobian in rigid regions is nearly orthogonal.

In a framework similar to some of the techniques referred above, we propose modelling a coordinate frame mapping using a dense deformation field and incorporating anatomical information. We explicitly enforce linear and rigid body constraints over large image areas as part of the deformation field regularization. As the optimization process iterates, regions under linear and rigid body constraints always move linearly or rigidly while unconstrained regions move under a free-form deformation. We demonstrate the approach using a modified Demons style algorithm, although the basic approach could be applied to many of the above techniques.

2. DEMONS

The image registration problem of two images is to estimate the transformation T that maps a position p_f in a fixed image I_f to the corresponding position $p_m = T(p_f)$ in the moving image I_m. Thirion [4] introduced the Demons registration algorithm to align images on the basis of optical flow. In the Demons algorithm, an iterative solution for T is given by $T_{i+1} = \partial T_i \circ T_i$, where ∂T_i is an incremental transformation based on a set of positions $p_{f_1}, \cdots, p_{f_N}$ and a set of incremental displacements $\vec{f}(p_{f_1}), \cdots, \vec{f}(p_{f_N})$,

$$\partial T_i = F\left(p_{f_1}, \cdots, p_{f_N}, \vec{f}(p_{f_1}), \cdots, \vec{f}(p_{f_N})\right), \quad (1)$$

$$\vec{f}(p_f) = \frac{(I_m(T_i(p_f)) - I_f(p_f))\nabla I_f(p_f)}{\|\nabla I_f(p_f)\|^2 + (I_m(T_i(p_f)) - I_f(p_f))^2} \quad (2)$$

The function $F()$ in equation 1 allows a diverse set of incremental transformations ∂T_i to be modelled, ranging from free-form deformations to parameterized transformations. In the former, a dense set of incremental displacement vectors is estimated. This is how the Demons algorithm is implemented in the Insight Segmentation and Registration Toolkit [5]. In the latter, parameterized transforms, such as affine or B-spline transformations, are estimated from the positions and incremental displacement vectors. This latter approach makes the Demons algorithm similar to parameter estimation approaches to image registration. The function $F()$ can also be used to regularize the displacement field.

In this paper, we explore using transformations T_i and incremental transformations ∂T_i whose representation and degrees of freedom vary spatially. Here, some positions in I_f will be governed by rigid body parameterized transformations while other positions in I_f will be governed by free-form deformations. This approach can be applied to other algorithms with a similar formulation (e.g. the Level-set motion registration by Vemuri *et al.* [6])

3. SPATIAL CONSTRAINTS ON DEFORMATION DEGREES OF FREEDOM

Our approach is to delineate regions, Ω_j, in the fixed image I_f and assign a transformation model Θ_j to each region. Region delineations can be derived from an atlas, can be the result of automated and semi-automated segmentation algorithms applied to individual structures, or defined manually. When registering CT imagery without injected contrast, large rigid bone structures can be easily identified using simple thresholding techniques followed by connected component techniques to isolate individual structures.

For each region, we prescribe a transformation model to govern its motion. Currently, we provide rigid, linear (affine), and free-form transformations. Other parameterized and spline based transformations are possible. The transformation model for each region is used to regularize the deformation field in a Demons style algorithm. Note that this approach can be applied to any formulation similar to the Demons algorithm. The registration algorithm is as follows:

1. Initialize deformation field, $T_0 \leftarrow \vec{0}$
2. Estimate incremental displacements,
$$\vec{f}(p_f) = \frac{(I_m(T_i(p_f)) - I_f(p_f))\nabla I_f(p_f)}{\|\nabla I_f(p_f)\|^2 + (I_m(T_i(p_f)) - I_f(p_f))^2} \quad (3)$$
3. Composite the incremental displacements into the current deformation field, $T_{i+1} = T_i + \partial T_i$
4. Regularize each region T_{i+1,Ω_j} based on its transformation model Θ_j
   (a) $\Theta_j \in$ free-form deformation:
$$T_{i+1,\Omega_j} \leftarrow T_{i+1,\Omega_j} * N(0, \Sigma) \quad (4)$$
   (b) $\Theta_j \in$ linear, $\Theta_j \in$ rigid:
$$\Theta_j^* = \text{argmin}_{\Theta_j} \|F(T_{i+1,\Omega_j}; \Theta_j) - T_{i+1,\Omega_j}\|^2 \quad (5)$$
$$T_{i+1,\Omega_j} \leftarrow F\left(T_{i+1,\Omega_j}; \Theta_j^*\right), \quad (6)$$
   where T_{i+1,Ω_j} is the portion of the deformation field under the region Ω_j.
5. Repeat with step 2

In the regularization step 4, we use the standard gaussian smoothing approach to yield a smoothly varying deformation field for the regions Ω_j whose transformation models Θ_j are free-form transformations. For other transformation types, we estimate the parameters of the transformation Θ_j that best describes (under an appropriate metric) region Ω_j's deformation field T_{i,Ω_j}. Then, we project T_{i,Ω_j} onto the space spanned by the transformation Θ_j. We then continue with Demons iterations until convergence. Note, that at each iteration, the deformation field is explicitly constrained to the feasible region prescribed by the region delineations Ω_j and the prescribed transformation types Θ_j. Thus, regions that are defined to move rigidly will move rigidly during the entire optimization process, yielding valid deformations at each iteration. This is in contrast to other techniques, where the rigidity constraints may only be fully satisfied at the optimum. The transformation estimation and projection operations are described in the following sections.

4. ESTIMATING LINEAR TRANSFORMATIONS

A position p_m in I_m is related to a position p_f in I_f through a transformation as well as through a displacement vector: $p_m = T(p_f)$, $p_m = p_f + \vec{f}(p_f)$. Combining these equations, each displacement vector can be written as a function of the transformation $\vec{f}(p_f) = T(p_f) - p_f$. If T is a linear transformation, $\vec{f}(p_f)$ can be written in matrix form $\vec{f}(p_f) = B \begin{bmatrix} p_f \\ 1 \end{bmatrix}$. Given a set of measurement locations p_{f_i} and a set of displacement vectors $\vec{f}(p_{f_i})$, we construct the linear multivariate model

$$Y = XB^T + U \quad (7)$$

where X and Y are matrices whose rows are composed of the positions $p_{f_1}, \cdots, p_{f_N}$ and the displacement vectors $\vec{f}(p_{f_1})$, $\cdots$, $\vec{f}(p_{f_N})$

$$X = \begin{bmatrix} p_{f_1}^T & 1 \\ \vdots & \vdots \\ p_{f_N}^T & 1 \end{bmatrix} \qquad Y = \begin{bmatrix} \vec{f}(p_{f_1})^T \\ \vdots \\ \vec{f}(p_{f_N})^T \end{bmatrix} \quad (8)$$

with the matrix U modelling the noise. When the noise is multivariate normal, the maximum likelihood estimator for B is given by [7]

$$B^T = \left(X^T X\right)^{-1} X^T Y. \quad (9)$$

To constrain the original set of displacement vectors to the space spanned by the estimated linear transformation, the vectors $\vec{f}(p_{f_i})$ are replaced with

$$\hat{\vec{f}}(p_{f_i}) = B \begin{bmatrix} p_{f_i} \\ 1 \end{bmatrix}. \quad (10)$$

We use equation 9 as the implementation of equation 5, estimating the parameters Θ_j of the linear transformation for a region Ω_j prescribed to move linearly. We use equation 10 as the implementation of equation 6, projecting the displacement vectors T_{i,Ω_j} onto this space, thereby enforcing the linear constraints.

Equation 9 provides the maximum likelihood estimate of the linear model, assuming all measurements are trusted equally. If the displacement vectors have varying certainty, a weighted estimator can also be used, where equations 7 and 9 become

$$WY = WXB^T + WU, \tag{11}$$

$$B^T = \left(X^T W^T W X\right)^{-1} X^T W^T W Y. \tag{12}$$

Equation 12 provides a mechanism to employ robust loss functions and M-estimators [8] to estimate the parameters of the linear model. Using M-estimators will increase the robustness of the linear model estimated from the displacement vectors.

5. ESTIMATING RIGID TRANSFORMATIONS

The linear model B estimated in equation 9 maps positions in I_f to displacement vectors that model an underlying affine motion of the positions. To model rigid body motion, additional constraints are needed on the parameters of the transformation. We modify the problem formulation from the linear case such that the goal is to estimate the rigid body transform (R, t) that maps positions in I_f to positions in I_m under rigid body motion. Here, equation 5 becomes

$$R^*, t^* = \text{argmin}_{R,t} \|Y - XR - t\|^2 \tag{13}$$

$$\text{subject to :} \quad R^T R = I \tag{14}$$

where the matrix X in this case has dropped the final column of 1's and the matrix Y contains positions $p_{f_i} + \vec{f}(p_{f_i})$ in I_m instead of the displacement vectors. A region of the deformation field Ω_j constrained by rigid motion is regularized by solving equation 13 under the constraints 14 and projecting the vectors $\vec{f}(p_{f_i})$ in Ω_j with $\hat{\vec{f}}(p_{f_i}) = Rp_{f_i} + t - p_{f_i}$. The constrained optimization problem in equations 13 and 14 operates in the Special Euclidean Group, SE(3). Horn [9] presents a closed form solution which changes the parametrization of the problem to use quaternions. The change of parametrization converts the optimization problem in equations 13 and 14 to a well known eigenvector problem. Several extensions to Horn's basic method have been proposed as the solution step iterated in Iterative Closest Point (ICP) registration [10, 11]. Many of these extensions can also be applied here, ignoring the iteration of ICP. In particular, Fitzgibbon [10] presents a formulation whereby robust loss functions can be incorporated into the metric, thereby increasing the robustness of the estimated rigid model to the displacement vectors.

6. RESULTS

We have tested our algorithm on both $2D$ and $3D$ real images. We compare the Demons registration algorithm from the Insight Segmentation and Registration Toolkit (ITK) with a version of this algorithm modified to enforce linear and rigid body constraints. A similar experiment was also performed using the Level-set motion registration algorithm from ITK for the $3D$ images.

Fig. 2 illustrates this technique applied to a $2D$ CT image of the cervical region of the spine. The ground truth coordinate transformation between the image sets is a translation of $3mm \times 3mm \times 3mm$. The registration without constraints fails to align the complete rib or vertebrae whereas the registrations incorporating linear and rigid body constraints align the data well (Fig. 2).

For the $3D$ case, we performed registrations on respiratory gated CT images that were obtained from 4 different patients. Each patient data comprised of 6 respiratory-gated volumes that were then registered to a chosen reference gate. In total, we tested each algorithm on 20 registrations $(5$ pairs$\times 4$ patients$)$. For the 3D volumes, the visual overlays (Fig. 3) show that the proposed approach minimizes unnatural warping of rigid structures while improving overlap with the reference image. To reinforce the visual assessments, we calculate Dice's Coefficient (DC) that measures the similarity between the bone-masks obtained from the algorithms with that obtained from the reference image. For binary masks A and B, $DC(A, B) = \frac{2|A \cap B|}{|A|+|B|}$. We observe a consistent improvement in the Dice's coefficient by using our method. Fig. 1 shows the results for a patient [5 registrations].

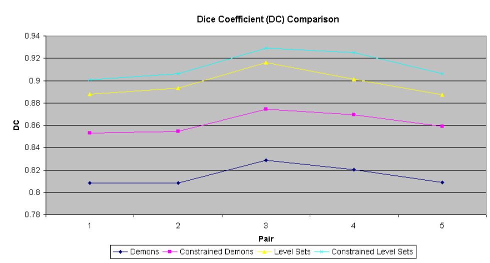
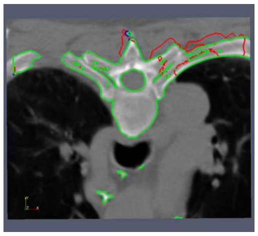


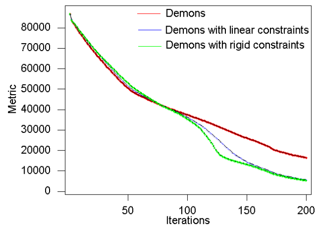
Fig. 1. DC between the bone-masks for patient #1: improvement in overlap using the proposed approach with both, Demons and Level-set motion methods

7. CONCLUSIONS

Deformable registration techniques are capable of modeling a wide range of nonrigid motion. However, when regions of an image are known to move rigidly, these techniques may not estimate true rigid motion. While deformable techniques can model rigid body vector fields, intermediate results before convergence and local minima results may not reflect realizable configurations. We have shown how a deformable registration technique known for estimating high degree of



(a)



(b)

Fig. 2. (a) Comparison of the contour of the constrained region. (b) Comparison of mean square metric convergence. Red: Demons, Blue: Linear constraints, Green: Rigid constraints [note: rigid and linear contours coincide in this case]

freedom deformations (Demons) can be constrained to incorporate rigid and linear motions embedded in free-form deformations. We achieve this by applying custom regularizations to each image region. The regularizations for linearly and rigidly constrained regions estimate a low parameter model of the motion from the current Demons vectors and use the estimate to project the deformation field in that region to satisfy the constraints. This allows constraints to be applied to large image regions and ensures that the constraints are satisfied throughout the Demons iterations. As shown in the $3D$ experiment, the approach can easily be extended to similar algorithms like Level-set motion registration. The low parameter models can be standard maximum likelihood estimates or weighted estimates using robust loss functions. This approach has also shown a benefit in the rate of convergence (Fig. 2(b)).

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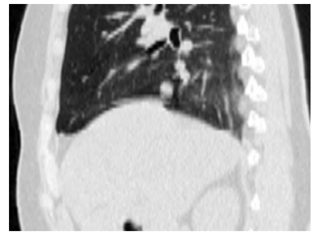
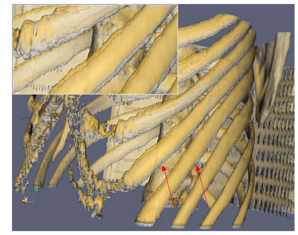
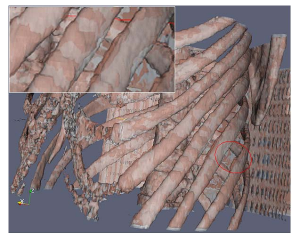


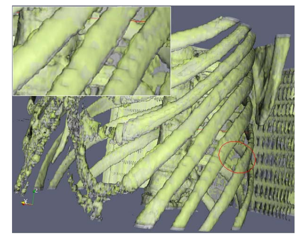
(a) Original CT Image



(b) Rigid registration



(c) Demons registration



(d) Constrained-Demons

Fig. 3. Post-registration overlays of warped (colored) to reference (gray) bone surfaces with the inset showing a magnified view. (b) Rigid: low deformation, low overlap (c) Demons: high deformation, high overlap (d) Constrained-Demons: low deformation, high overlap

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